The effect of atomic excitation on the shape of the ^{3}H beta spectrum near the end point

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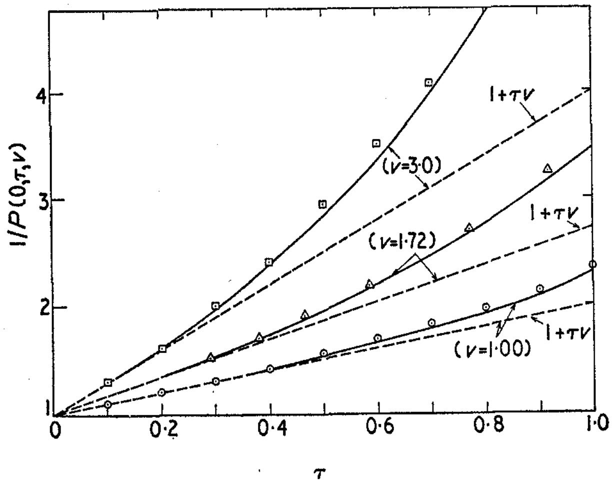

Figure 2. Theoretical and experimental plots of $1/P(0, \tau, v)$ against τ. ⊙ Measured for $v = 1{\cdot}00$; △ measured for $v = 1{\cdot}72$; □ measured for $v = 3{\cdot}00$.

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Bedard, G., 1967, *Phys. Rev.*, **161**, 1304–8.
Glauber, R. J., 1965, *Quantum Optics and Electronics*, eds C. DeWitt, A. Glandin and C. Cohen (New York: Gordon and Breach), p. 178.
Jakeman, E., Oliver, C. J., and Pike, E. R., 1968, *J. Phys.* A: *Gen. Phys.*, **1**, 406–8.
—— 1970, *J. Phys.* A: *Gen. Phys.*, **3**, L55–9.
Jakeman, E., and Pike, E. R., 1968, *J. Phys.* A: *Gen. Phys.*, **1**, 128–38.
Kelly, H. C., 1972, *J. Phys.* A: *Gen. Phys.*, in the press.

The effect of atomic excitation on the shape of the ^{3}H beta spectrum near the end point

Abstract. The effect of atomic excitation on the shape of the ^{3}H beta spectrum near the end point is calculated and found to be small. However, depending on the type of spectrometer and the method of analysis of data, the effect could lead to a systematic error which is not negligible compared with the accuracies quoted in some recent determinations of the end point.

The beta decay of ^{3}H is the simplest allowed decay, apart from the decay of the free neutron, and accurate study of the spectrum near the end point is important for several reasons. The ft value is strongly dependent upon the value of the end point energy (Bahcall 1966) and an accurate value of ft is required to check the predictions of theory concerning mesonic exchange effects (Blin-Stoyle and Tint 1967). The spectrum shape near the end point also gives an upper limit for the neutrino rest mass. Recently there have appeared some very accurate determinations of the spectrum end point and of the neutrino rest mass, and the results are summarized in table 1. Bergkvist (1969) does not give a value of the end point energy but the result

Table 1. The results of some recent measurements of the ^{3}H beta spectrum

End point energy (keV)	ft (s)	m_ν (eV)	Reference
18.72 ± 0.05	1159 ± 11	<200	Salgo and Staub (1969)
18.57 ± 0.075	1130 ± 15	<75	Daris and St.-Pierre (1969)
18.63 ± 0.05	1143 ± 11	<60	Bergkvist (1969)
18.54 ± 0.095	1144 ± 9	—	Lewis (1970)

is given by Blin-Stoyle (1969, private communication). The measurements were made using magnetic spectrometers, with the exception of the work of Lewis (1970) where a solid state detector was used, and the results agree reasonably well considering the difficulties of using solid sources and of overcoming the (very considerable) problem of background near the end point.

When measurements are quoted to the accuracies given in table 1 it seems reasonable to discuss the effects of higher order corrections. The radiative corrections have been considered by Salgo and Staub (1970) and by Jarlskog (1969), who concludes that the shape of the spectrum near the upper limit and values deduced for the neutrino rest mass are not altered by this effect since the result is to increase the transition rate by a constant amount all along the spectrum. There is still some doubt as to the precise nature of the radiative corrections (Blin-Stoyle and Freeman 1970, Wilkinson and Macefield 1970, Halpern 1970) but the problem is probably not serious for such a very low Z decay as that of ^{3}H. It is the purpose of this letter to estimate the effect of atomic excitation and ionization on the spectrum shape and end point.

The atomic excitation following beta decay was first calculated by Feinberg (1941) and Migdal (1941) and there has recently been theoretical (Crasemann and Stephas 1969) and experimental (Fischbeck *et al.* 1971, der Mateosian 1971) interest in the subject. The excitation or ionization (shake-off) is attributed to the sudden change of nuclear charge during the beta decay and can be calculated on the assumption that the beta particle usually has a much greater velocity than the atomic electrons so that direct interaction between them can be ignored. Electron shake-off is an effect additional to the single ionization required by conservation of charge in the beta decay process. Beta decay is a property of the entire atom and the difference in binding energy between the initial and final atomic states makes a contribution to the total energy available for beta decay, denoted by W_0. W_0, in this case, is the mass difference between a neutral ^{3}H atom having its atomic electron in the 1S state and a singly charged ^{3}He ion with the atomic electron in the 1S state. If the atomic electron of ^{3}He ends up in anything other than the 1S state then there will be some amount of energy less than W_0 available for sharing between the beta particle and the neutrino.

In most cases the probability of excitation of a K-shell electron into the continuum is much greater than the probability of excitation to a bound state since many of these are filled and the exclusion principle operates, but this is not the case for the ^{3}H decay as there is only one orbital electron involved.

The early calculations (Feinberg 1941, Migdal 1941) assumed that excitation, or shake-off, was independent of the energy of the nuclear beta particle but in general this is not so since the available energy is being split among three particles and the phase space factor must be modified. However, the present calculation shows that most of the excitation occurring in the ^{3}H decay takes place to bound states of ^{3}He and it seems reasonable to assume that the only effect this has is to reduce slightly the amount of energy available for sharing between the beta particle and the neutrino. On this assumption, then, and with energy independent matrix elements, the ^{3}H beta spectrum has the form

$$P(W)\,\mathrm{d}W \propto \sum_i a_i F(Z, W) pW(W_0 - \delta_i - W)^2\,\mathrm{d}W.$$

Here, a_i is the probability of the atomic electron being in the ith atomic state of ^{3}He after the decay and δ_i represents the energy difference between the 1S state and the ith atomic state of ^{3}He.

Calculation of a_i was performed by evaluating the overlap integral between the 1S state of ^{3}H and the various S states of singly ionized ^{3}He using nonrelativistic hydrogen-like wavefunctions. The results, given in table 2, were checked against the general formula of Migdal (1941) and it appears that there is a misprint in equation (6) where the right hand side should be multiplied by a factor Z_1/Z.

Table 2. The results of the calculation of the overlap integrals

^{3}He state	a	δ (eV)
1S	0·702	0
2S	0·25	41
3S, 4S ... etc.	$\simeq$ 0·018	50
Continuum	$\simeq$ 0·03	100

The composite beta spectrum was constructed according to the above recipe using a value of 18·57 keV for the end point energy. The term $F(Z, W)$ is constant over the restricted energy range considered. Some 3% of decays lead to states in the continuum and theory (Migdal 1941) shows that the shake-off electron is most likely to have kinetic energy of the order of the K-shell binding energy. The energy deficit is therefore uncertain, but the calculated spectrum is not sensitive to the value assumed (for instance a value of 70 eV gives virtually the same result).

Assuming zero neutrino rest mass the Fermi–Kurie plots of the composite spectrum and a pure spectrum (ie with no excitation or shake-off) were constructed and are shown in figure 1. The two plots have the same end point but the plot derived from the composite spectrum is nonlinear close to the end point. It would be very difficult experimentally to obtain measurements with sufficient statistical accuracy to see such an effect and presumably the linear part of the plot would be extrapolated to give an end point some 12 eV too low. The question as to whether or not any particular experiment is prone to such a systematic error depends on the type of detector used and the method of analysis of the results. If a magnetic spectrometer is used then the energy of excitation would certainly go undetected and if an upper limit for the neutrino rest mass is deduced from the spectrum shape and it is assumed to be zero for the purpose of drawing Fermi–Kurie plots, then the effect should be considered as a possible source of systematic error. If a retarding field spectrometer

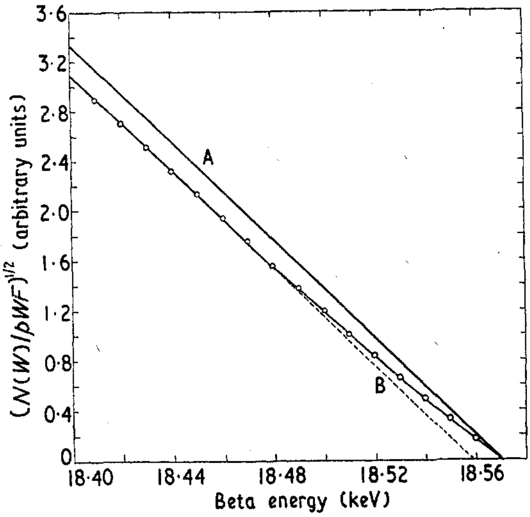

Figure 1. Fermi–Kurie plots for an end point energy of 18·57 keV. The plots are for a 'pure' spectrum (ie with no excitation or shake-off) and a composite spectrum constructed as explained in the text. Curve A, linear plot for 'pure' spectrum. Curve B, nonlinear plot for composite spectrum showing false end point.

is used, or a proportional counter, or a calorimeter, the effect can be ignored. Finally, the effect is small and would only make a difference of about two units in the *ft* value, or in the uncertainty in that value, but reference to table 1 shows that the effect is not completely negligible.

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Bahcall, J. N., 1966, *Nucl. Phys.*, **75,** 10–6.
Bergkvist, K. E., 1969, *Topical Conference on Weak Interactions, Geneva* 1969 (Geneva: CERN), pp. 91–100.
Blin-Stoyle, R. J., 1969, *Phys. Rev.*, **188,** 1540–1.
Blin-Stoyle, R. J., and Freeman, J. M., 1970, *Nucl. Phys.* A, **150,** 369–78.
Blin-Stoyle, R. J., and Tint, M., 1967, *Phys. Rev.*, **160,** 803–8.
Crasemann, B., and Stephas, P., 1969, *Nucl. Phys.* A, **134,** 641–54.
Daris, R., and St.-Pierre, C., 1969, *Nucl. Phys.* A, **138,** 545–55.
Feinberg, E. L., 1941, *J. Phys.-USSR*, **4,** 423–38.

Fischbeck, H. J., Wagner, F., Porter, F. T., and Freedman, M. S., 1971, *Phys. Rev.* C, **3**, 265–74.
Halpern, T. A., 1970, *Phys. Rev.* C, **1**, 1928–38.
Jarlskog, C., 1969, *Ark. Fys.*, **40**, 509–12.
Lewis, V. E., 1970, *Nucl. Phys.* A, **151**, 120–8.
der Mateosian, E., 1971, *Phys. Rev.* A, **3**, 573–7.
Migdal, A., 1941, *J. Phys.-USSR*, **4**, 449–53.
Salgo, R. C., and Staub, H. H., 1969, *Nucl. Phys.* A, **138**, 417–28.
Wilkinson, D. H., and Macefield, B. E. F., 1970, *Nucl. Phys.* A, **158**, 110–6.

Elastic scattering of electrons by solar neutrinos and weak interaction theories

Abstract. The counting rate of recoil electrons for the elastic scattering of electrons by solar neutrinos, considering that the CNO cycle is responsible for nuclear energy generation in the sun, is calculated according to the photon–neutrino weak coupling theory as well as the current–current coupling theory. It is suggested that this result can be used to decide which type of weak interaction exists in nature.

Recently from the energy balance between the nuclear energy generation and the neutrino emission according to the photon–neutrino weak coupling theory it was shown by Ray Chaudhuri (1971) that the sun derives its nuclear energy from the CNO cycle. So the neutrinos from the pp cycle energy generation (Bahcall 1967)

$$
\begin{array}{lll}
{}^{1}\mathrm{H}+{}^{1}\mathrm{H}+\mathrm{e}^{-} \to {}^{2}\mathrm{H}+\mathrm{e}^{+}+\nu_{\mathrm{e}} & (E_{\nu,\max} = 0{\cdot}420\ \mathrm{MeV}) & \\
{}^{2}\mathrm{H}+{}^{1}\mathrm{H} \to {}^{3}\mathrm{He}+\gamma & {}^{3}\mathrm{He}+{}^{3}\mathrm{He} \to {}^{4}\mathrm{He}+2{}^{1}\mathrm{H} & \\
{}^{3}\mathrm{He}+{}^{4}\mathrm{He} \to {}^{7}\mathrm{He}+\gamma & \mathrm{e}^{-}+{}^{7}\mathrm{He} \to {}^{7}\mathrm{Li}+\nu_{\mathrm{e}} & (E_{\nu,\max} = 0{\cdot}861\ \mathrm{MeV}) \\
\qquad \to {}^{7*}\mathrm{Li}+\nu_{\mathrm{e}} & (E_{\nu,\max} = 3{\cdot}383\ \mathrm{MeV}) & \\
{}^{7}\mathrm{Li}+{}^{1}\mathrm{H} \to 2{}^{4}\mathrm{He} & {}^{3}\mathrm{He}+{}^{4}\mathrm{He} \to {}^{7}\mathrm{He}+\gamma & {}^{7}\mathrm{He}+{}^{1}\mathrm{H} \to {}^{8}\mathrm{H}+\gamma \\
{}^{8}\mathrm{H} \to {}^{8*}\mathrm{He}+\mathrm{e}^{+}+\nu_{\mathrm{e}} & (E_{\nu,\max} = 14{\cdot}06\ \mathrm{MeV}) &
\end{array}
$$

should not be taken to be responsible for the observed counting rate for the solar neutrino experiment of Reines and Kropp (1964). Therefore only neutrinos from CNO cycle energy generation (Bahcall 1967)

$$
\begin{array}{lll}
{}^{12}\mathrm{C}+{}^{1}\mathrm{H} \to {}^{13}\mathrm{N}+\gamma & {}^{13}\mathrm{N} \to {}^{13}\mathrm{C}+\mathrm{e}^{+}+\nu_{\mathrm{e}} & (E_{\nu,\max} = 1{\cdot}20\ \mathrm{MeV}) \\
{}^{13}\mathrm{C}+{}^{1}\mathrm{H} \to {}^{14}\mathrm{N}+\gamma & {}^{14}\mathrm{N}+{}^{1}\mathrm{H} \to {}^{15}\mathrm{O}+\gamma & \\
{}^{15}\mathrm{O} \to {}^{15}\mathrm{N}+\mathrm{e}^{+}+\nu_{\mathrm{e}} & & (E_{\nu,\max} = 1{\cdot}74\ \mathrm{MeV}) \\
{}^{15}\mathrm{N}+{}^{1}\mathrm{H} \to {}^{12}\mathrm{C}+{}^{4}\mathrm{He} & &
\end{array}
$$

are mainly responsible for the observed counting rate of the solar neutrino experiment of Davis *et al.* (1968) and the solar neutrino experiment of Reines and Kropp (1964) should be adjusted along this line. Assuming a certain lepton–hadron relation proposed by Bandyopadhyay (1964) (the configuration for the neutron and proton was taken as $\mathrm{n} = \mu^{-}\mathrm{B}^{+}$ and $\mathrm{p} = \nu_{\mu}\mathrm{B}^{+}$, where B is the baryonic matter) and the dynamical origin of charge (Bandyopadhyay 1970), Bandyopadhyay (1971, submitted for